



Mathematics Calculation Policy 2021-22

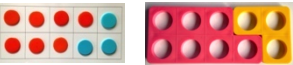
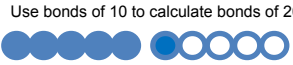


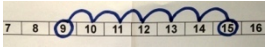
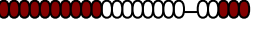
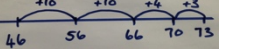

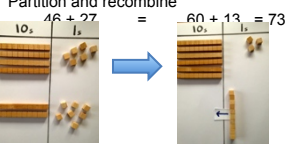

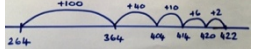
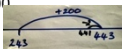


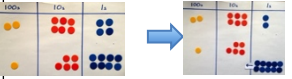





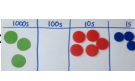
Summary

This calculation policy has been devised to support all staff at Stepping Stones School in understanding the expectations of the National Curriculum. It will ensure a whole school understanding of when a child should learn new calculation concepts in alignment with their mathematical development.

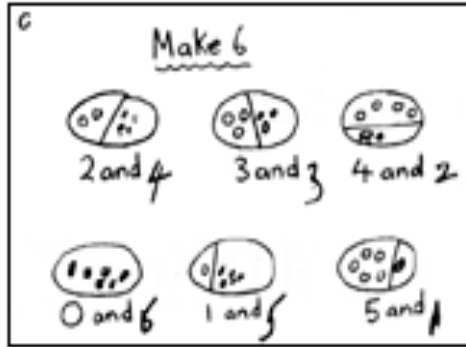
Principles

- At Stepping Stones we are focused on developing proficiency with the expected formal written methods for the end of Year 6. The progression guidance for each operation (detailed in this policy) is designed to flow into these expected methods.
- We acknowledge that our children come from a variety of schools and, as such, upon entering our school, discussions will be held with the pupils and/or their mainstream schools to ascertain the calculation methods they have been taught/are confident using.
- This policy provides examples of a variety of mental and written calculation methods for each operation. These methods are drawn from the AET Mathematics Calculation Policy (credit to Academies Enterprise Trust) and the Lancashire Calculations Policy.
- There is an extended version of the Lancashire Calculations Policy for each operation on our school server to be accessed for further support/clarification/exemplars. This should be used alongside the LAPs document.
- This policy suggests specific practical equipment and approaches for each year group to support our pupils in developing the conceptual understanding that will enable them to move more rapidly and efficiently towards the formal written methods expected.
- Many of our children are working below or significantly below their age related expectations. Children will be taught and supported in using the calculation methods for the age related expectations year group they are currently working within and not their chronological age year group. Through this differentiated approach teachers will be able to use methods and materials from earlier year groups to bridge any gaps in a child's understanding.
- The 'Written Methods', 'With jottings ... or in your head' and 'Just know it' sections list the National Curriculum expectations for each year group for calculation.
- The 'Foundations' section for each year group highlights the skills and knowledge that should be addressed on a regular basis within this year group to ensure that children have the necessary fluency to address the new approaches required.

Addition

Written Methods	Read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs	Add and subtract two two-digit numbers using concrete objects, pictorial representations progressing to formal written methods	Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction	Add and subtract numbers with up to 4 digits using the formal written methods of columnar addition where appropriate	Add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)	Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
<p>Developing conceptual understanding</p> <p>Number bonds  (Ten frame) Numicon</p> <p>Use bonds of 10 to calculate bonds of 20 </p> <p>Count all </p> <p>Count on </p> <p>Count on, on number track, in 1s </p>	<p>Number track / Number line – jumps of 1 then efficient jumps using number bonds $18 + 5 = 23$ </p> <p>$46 + 27 = 73$ Count in tens then bridge. </p> <p>25 + 29 by + 30 then - (Round and adjust) </p> <p>Partition and recombine $46 + 27 = 73$ </p> <p>$24 + 10 = 34$ $+ 10 = 44$ $+ 10 = 54$ </p>	<p>Number line: $264 + 158$ efficient jumps </p> <p>$40 + 80 = 120$ using $4 + 8 = 12$ So $400 + 800 = 1200$ </p> <p>$243 + 198$ by +200 then -2 (Round and adjust) </p> <p>Pairs that make 100 $23 + 77$ </p> <p>Place value counters, 100s, 10s, 1s $264 + 158$ </p> <p>422 (Also with £, 10p and 1p) </p>	<p>Place Value Counters $2458 + 596$ Show 2458 and 596 </p> <p>Combine the 1s. Exchange ten 1s for a 10 counter. </p> <p>Combine the 10s. Exchange ten 10s for a 100 counter. </p> <p>Combine the 100s. Exchange ten 100s for a 1000 counter </p> <p>Read final answer Three thousand and fifty-four. </p>	<p>Set out the calculation In columns. $\begin{array}{r} 23454 \\ + 596 \\ \hline \end{array}$</p> <p>Find the sum of the ones. $4 \text{ ones} + 6 \text{ ones} = 10 \text{ ones}$ (or 1 ten and 0 ones) so record 0 in the ones and 1 below the line in the tens.</p> <p>Find the sum of the tens. $5 \text{ tens} + 9 \text{ tens} + 1 \text{ ten} = 15 \text{ tens}$ (or 1 hundred and 5 tens) so record a 5 in the tens and 1 below the line in the hundreds.</p> <p>Find the sum of the hundreds. $4 \text{ hundreds} + 5 \text{ hundreds} + 1 \text{ hundred} = 10 \text{ hundreds}$ (or 1 thousand and 0 hundreds) so record a 0 in the hundreds and a 1 in the thousands.</p> <p>Find the sum of the thousands. $3 \text{ thousands} + 1 \text{ thousand} = 4 \text{ thousands}$ so record a 4 in the thousands column.</p> <p>Find the sum of the ten thousand There are only 2 ten thousands so record a 2 in the final column $\begin{array}{r} 23454 \\ + 596 \\ \hline 24050 \\ \hline \end{array}$</p>	<p>Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why</p> <p>Add and subtract numbers mentally with increasingly large numbers</p>	<p>Perform mental calculations, including with mixed operations and large numbers</p>
<p>With jottings</p> <p>... or in your head</p>	<p>Solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as $7 = \quad - 9$</p>	<p>Add and subtract numbers using concrete objects, pictorial representations, and mentally, including:</p> <ul style="list-style-type: none"> * a two-digit number and ones * a two-digit number and tens * two two-digit numbers * adding three one-digit numbers 	<p>Add and subtract numbers mentally, including:</p> <ul style="list-style-type: none"> * a three-digit number and ones * a three-digit number and tens * a three-digit number and hundreds 	<p>Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why</p>	<p>Add and subtract numbers mentally with increasingly large numbers</p>	<p>Perform mental calculations, including with mixed operations and large numbers</p>
<p>Just know it!</p>	<p>Represent & use number bonds and related subtraction facts within 20 Add and subtract one-digit and two-digit numbers to 20, including zero</p>	<p>Recall and use addition and subtraction facts within 20 Recall and use addition and subtraction facts within 20 fluently, and derive and use related facts up to 100</p>				
<p>Year</p>	<p style="text-align: center;">1</p>	<p style="text-align: center;">2</p>	<p style="text-align: center;">3</p>	<p style="text-align: center;">4</p>	<p style="text-align: center;">5</p>	<p style="text-align: center;">6</p>
<p>Foundations</p>	<p>1 more Number bonds: 5, 6 Largest number first. Number bonds: 7, 8 Add 10. Number bonds: 9, 10 Ten plus ones. Doubles up to 10 Use number bonds of 10 to derive bonds of 11</p>	<p>10 more Number bonds: 20, 12, 13 Number bonds: 14, 15 Add 1 digit to 2 digit by bridging. Partition second number, add tens then ones Add 10 and multiples. Number bonds: 16 and 17 Doubles up to 20 and multiples of 5 Add near multiples of 10. Number bonds: 18, 19 Partition and recombine</p>	<p>Add multiples of 10, 100 Add single digit bridging through boundaries Partition second number to add Pairs of 100 Use near doubles to add Add near multiples of 10 and 100 by rounding and adjusting Partition and recombine</p>	<p>Add multiples of 10s, 100s, 1000s Fluency of 2 digit + 2 digit Partition second number to add Decimal pairs of 10 and 1 Use near doubles to add Adjust both numbers before adding Add near multiples Partition and recombine</p>	<p>Add multiples of 10s, 100s, 1000s, tenths, Fluency of 2 digit + 2 digit including with decimals Partition second number to add Use number facts, bridging and place value Adjust numbers to add Partition and recombine</p>	<p>Add multiples of 10s, 100s, 1000s, tenths, hundredths Fluency of 2 digit + 2 digit including with decimals Partition second number to add Use number facts, bridging and place value Adjust numbers to add Partition and recombine</p>

Stage 1 - EYFS



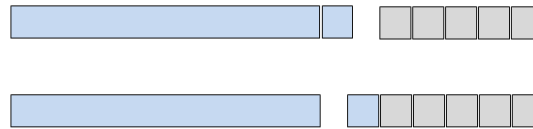
Children are taught that addition is the combining of two or more amounts. They begin by counting all of the items in the groups, then move on to counting on from the largest amount. Children are encouraged to develop a mental image of the size of numbers. They learn to think about addition as combining amounts in practical, real life situations. They begin to record addition number sentences such as $2 + 4 = 6$ and $8 = 3 + 5$ and $3 + 2 + 4 = 9$

Stage 2 – Year 1

Children move on to using Base 10 equipment to support their developing understanding of addition.

$11 + 5 = 16$

11 cubes are lined up (1 ten and 1 unit/one).
5 cubes are added to the line of 11 giving a total of 16.



If possible, use two different colours of base 10 equipment so that the initial amounts can still be seen.

Stage 3 – Year 2

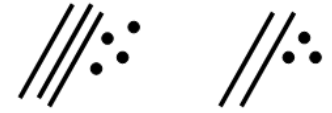
Children continue to use the Base 10 equipment to support their calculations, including exchanging 10 units/ones for 1 ten when the total of the units/ones is 10 or more. They will record their own drawings of the Base 10 equipment, using lines for 10 rods and dots for the unit blocks.

$34 + 23 = ?$

The units/ones are added first $4 + 3 = 7$

The tens are added next $30 + 20 = 50$

Both answers are put together $50 + 7 = 57$

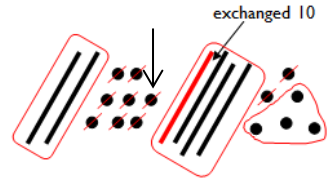


$28 + 36 = ?$

The units/ones are added first

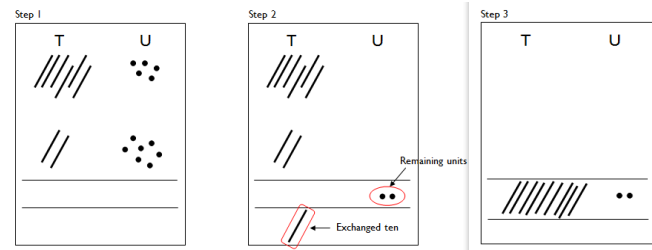
$8 + 6 = 14$ with ten units/ones exchanged for 1 ten.

A ring is put around the units/ones not exchanged – this is the units part of the answer. The tens are then added, including the exchanged ten, to complete the sum.



Stage 4 – Year 3

$65 + 27$



Written method

Step 1	Step 2	Step 3																																								
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Stage 5 – Year 3-6

HTU			
$\begin{array}{r} 625 \\ + 48 \\ \hline 673 \\ 1 \end{array}$	$\begin{array}{r} 367 \\ + 85 \\ \hline 452 \\ 11 \end{array}$	$\begin{array}{r} 321 \\ + 7 \\ \hline 328 \\ 1 \end{array}$	$\begin{array}{r} \pounds 3.48 \\ + \pounds 0.78 \\ \hline \pounds 4.26 \\ 11 \end{array}$

This is the final stage of the method, and should be continued to be used for all written addition calculations.

The example top left would be 'said' as follows:
 $5 + 8 = 13$, put 3 down and carry the 10
 $20 + 40 + 10$ that was carried over = 70 (7 written in the tens column)
 $600 + 0 = 600$ (6 written in the hundreds column)

Children will be expected to use this method for adding numbers with more than 3 digits, numbers involving decimals and adding any number of amounts together.

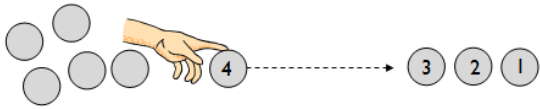
Children should not be made to go onto the next stage if:

- 1) they are not ready.
- 2) they are not confident.

Children should be encouraged to consider if a mental calculation would be appropriate before using written methods.

Stage 1 - EYFS

Children will subtract two numbers by taking one away from the other and counting how many are left.



Children are encouraged to develop a mental image of the size of numbers. They learn to think about subtraction as 'take away' in practical, real life situations.

They begin to record subtraction number sentences such as $8 - 5 = 3$



Stage 2 – Year 1

Children move on to using Base 10 equipment alongside a number track to support their developing understanding of subtraction.

$13 - 4 = ?$

13 cubes are lined up.

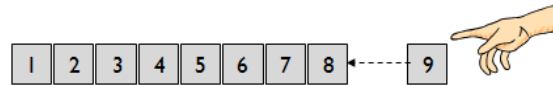
4 cubes are removed from the end of the line leaving 9 left. It is important that children keep track of how many have been removed.



Touch count and remove the number to be taken away.



Touch count to find the number that remains.



Stage 3 – Year 2

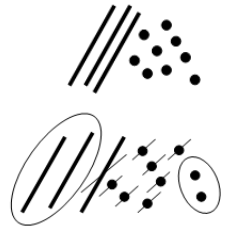
Children continue to use the Base 10 equipment to support their calculations. They will record their own drawings of the Base 10 equipment, using lines for 10 rods and dots for the unit blocks.

$39 - 17 = ?$

39 is drawn

17 is crossed out

A ring is drawn around what is left to give the answer giving 22



Step 1

$37 - 19 = ?$

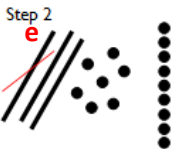
37 is drawn

9 units cannot be crossed out, so a ten is crossed out and exchanged for 10 ones which are in a line.

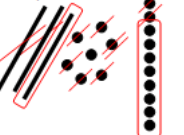
e is written next to the exchanged ten.

19 is crossed out

A ring is drawn around what is left to give the answer 18



Step 2



Step 3

Stage 4A – Year 3

$$\begin{array}{r} 89 \\ - 57 \\ \hline 30 \end{array} = \begin{array}{r} 80 \\ - 50 \\ \hline 30 \end{array} \rightarrow \begin{array}{r} 9 \\ - 7 \\ \hline 2 \end{array} = 32$$

The calculation should be read as subtract 7 from 9 or 9 subtract 7.

Children move from using the Base 10 method to expanded vertical method, using base 10 notation and arrow cards. Children learn to subtract the least significant digits first (start with the numbers on the right and work from right to left). The answer to each individual subtraction is written underneath before these answers are recombined.

Stage 4B – Year 3

This stage involves exchange.

It is clear that there are not enough units to subtract 6 from 1, so one of the tens from the 70 is exchanged for 10 ones.

The initial number 71 is rearranged as 60 and 11 to make the calculation easier.

This would be recorded by the children as:

$$\begin{array}{r} 70 \\ - 40 \\ \hline 30 \end{array} \rightarrow \begin{array}{r} 1 \\ - 6 \\ \hline 6 \end{array}$$

$$\begin{array}{r} 60 \\ - 40 \\ \hline 20 \end{array} \rightarrow \begin{array}{r} 11 \\ - 6 \\ \hline 5 \end{array} = 25$$

$$\begin{array}{r} 60 \\ - 40 \\ \hline 20 \end{array} \rightarrow \begin{array}{r} 11 \\ - 6 \\ \hline 5 \end{array} = 25$$

Stage 5 – Year 4

This final stage is the compact method of decomposition. The example shows how the same calculation would be carried out using the previous method and the final method.

$$\begin{array}{r} 754 \\ - 86 \\ \hline \end{array}$$

Stage 4B

$$\begin{array}{r} 600 \\ - 700 \\ \hline 600 \end{array} \rightarrow \begin{array}{r} 140 \\ - 50 \\ \hline 80 \end{array} \rightarrow \begin{array}{r} 14 \\ - 6 \\ \hline 8 \end{array} = 668$$





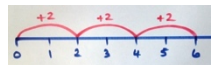
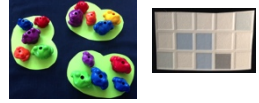
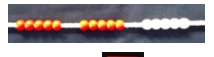
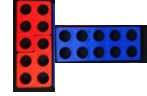



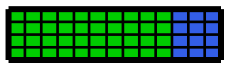
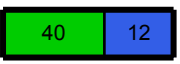

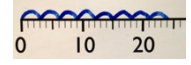
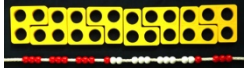
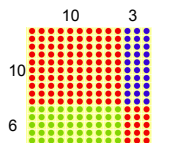

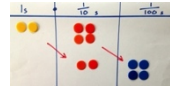
becomes

Stage 5 – Year 4-6

$$\begin{array}{r} 6141 \\ - 764 \\ \hline 668 \end{array}$$

This is the final stage of the process and will continue to be used with larger numbers and numbers involving decimals.

Multiplication

Written Methods		Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (x), division (÷) and equals (=) signs	Write and calculate mathematical statements for ÷ using the x tables they know progressing to formal written methods.	Multiply two-digit and three-digit numbers by a one-digit number using formal written layout	Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers	Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication																																																							
Developing conceptual understanding	<p>2 frogs on each lily pad.</p>     	<p>5 frogs on each lily pad $5 \times 3 = 15$</p>   <p>$5 \times 2 = 2 \times 5$</p>  <p>Build tables on counting stick</p>  <p>Link to repeated addition</p> 	<p>If I know $10 \times 8 = 80$ then ...</p>  <p>So $13 \times 4 = 10 \times 4 + 3 \times 4$</p>   <p>Build tables on counting stick</p>   	<p>43×6 by partitioning</p> <table border="1" data-bbox="1209 375 1433 478"> <tr> <td>X</td> <td>40</td> <td>3</td> </tr> <tr> <td>6</td> <td>240</td> <td>18</td> </tr> </table> <p>$43 \times 6 = 240 + 18 = 258$</p> <p>If I know $4 \times 6 = 24$ the 40×6 is ten times bigger.</p> <p>13×16 by partitioning</p>  <p>$100 + 30 + 60 + 18 = 208$</p> <p>Build tables on counting stick</p> 	X	40	3	6	240	18	<p>Grid method linked to formal written method</p> <table border="1" data-bbox="1523 391 1736 502"> <tr> <td>x</td> <td>200</td> <td>40</td> <td>3</td> </tr> <tr> <td>30</td> <td>6000</td> <td>1200</td> <td>90</td> </tr> <tr> <td>6</td> <td>1200</td> <td>240</td> <td>18</td> </tr> </table> <p>$7290 + 1458 = 8748$</p> <p>If I know 4×6 then 0.4×6 is ten times smaller 0.4×0.6 is ten times smaller again.</p> 	x	200	40	3	30	6000	1200	90	6	1200	240	18	<p>To multiply 5172 by 38 find the sum 5172×30 & 5172×8.</p> <p>5172×30: This is the same as $5172 \times 3 \times 10$. Therefore, record a 0 in the 1s column to take care of the 'ten times bigger' and begin to calculate 5182×3.</p> <table border="1" data-bbox="1881 574 2184 654"> <tr> <td>5172</td> <td>5172</td> <td>5172</td> <td>5172</td> <td>5172</td> </tr> <tr> <td>$\times 38$</td> <td>$\times 38$</td> <td>$\times 38$</td> <td>$\times 38$</td> <td>$\times 38$</td> </tr> <tr> <td>0</td> <td>60</td> <td>2160</td> <td>5160</td> <td>155160</td> </tr> </table> <p>Then calculate 5172 multiplied by 8 and record beneath:</p> <table border="1" data-bbox="1881 694 2184 790"> <tr> <td>5172</td> <td>5172</td> <td>5172</td> <td>5172</td> </tr> <tr> <td>$\times 38$</td> <td>$\times 38$</td> <td>$\times 38$</td> <td>$\times 38$</td> </tr> <tr> <td>155160</td> <td>155160</td> <td>155160</td> <td>155160</td> </tr> </table> <p>Finally add the two parts together:</p> <table border="1" data-bbox="1881 805 2184 917"> <tr> <td>5172</td> <td>5172</td> </tr> <tr> <td>$\times 38$</td> <td>$\times 38$</td> </tr> <tr> <td>155160</td> <td>155160</td> </tr> <tr> <td>41376</td> <td>41376</td> </tr> <tr> <td><u>196536</u></td> <td><u>196536</u></td> </tr> </table>	5172	5172	5172	5172	5172	$\times 38$	$\times 38$	$\times 38$	$\times 38$	$\times 38$	0	60	2160	5160	155160	5172	5172	5172	5172	$\times 38$	$\times 38$	$\times 38$	$\times 38$	155160	155160	155160	155160	5172	5172	$\times 38$	$\times 38$	155160	155160	41376	41376	<u>196536</u>	<u>196536</u>
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With jottings ... or in your head	Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher	Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot. Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts	Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental methods	Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers. Recognise and use factor pairs and commutativity in mental calculations	Multiply and divide numbers mentally drawing upon known facts. Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000. Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers establish whether a number up to 100 is prime	Perform mental calculations, including with mixed operations and large numbers																																																							
Just know it!	Count in multiples of twos, fives and tens	Recall and use x and ÷ facts for the 2, 5 and 10 x tables, including recognising odd and even numbers.	Recall and use x and ÷ facts for the 3, 4 and 8 times tables.	Recall x and ÷ facts for x tables up to 12 x 12.	Recall prime numbers up to 19 know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers. Recognise and use square numbers and cube numbers, and the notation for squared (²) and cubed (³)																																																								
Year	1	2	3	4	5	6																																																							
Foundations	Count in 2s	2 x table	Review 2x, 5x and 10x	4x, 8x tables 10 times bigger	4x, 8x tables 100, 1000 times bigger	Multiplication facts up to 12 x 12																																																							
	Count in 10s	10 x table	4x table	3x, 6x and 12x tables	3x, 6x and 12x tables 10, 100, 1000 times smaller	Partition to multiply mentally																																																							
	Doubles up to 10	Doubles up to 20 and multiples of 5	Double two digit numbers	Double larger numbers and decimals	Double larger numbers and decimals	Double larger numbers and decimals																																																							
	Count in 5s	5 x table	8 x table	3x, 9x tables	3x, 9x tables	Multiplication facts up to 12 x 12																																																							
	Double multiples of 10	Count in 3s	3 x table	11x, 7 x tables	11x, 7 x tables Partition to multiply mentally	Partition to multiply mentally																																																							

Stage 1 - EYFS

Children are encouraged to develop a mental image of the size of numbers. They learn to think about equal groups or sets of objects in practical, real life situations. They begin to record these situations using pictures.



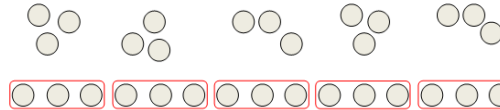
A child's jotting showing fingers on each hand as a double.



A child's jotting showing double three as three cookies on each plate.

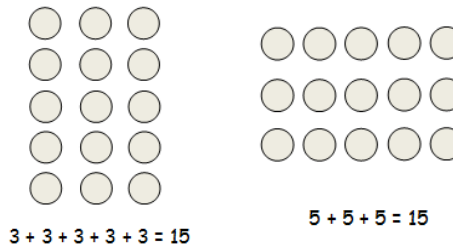
Stage 2 – Year 1 and Year 2

Children can then be introduced to the image of a rectangular array, initially through real items such as egg boxes, baking trays, ice cube trays, wrapping paper etc. and using these to show that counting up in equal groups can be a quicker way of finding a total. Children understand that multiplication is repeated addition and that can be done by counting in equal steps/groups.



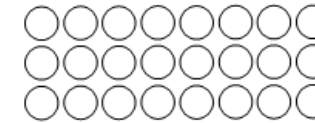
or

Children also understand that 3×5 is the same as 5×3

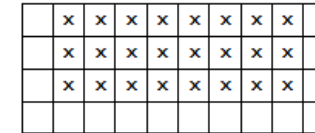


Stage 3 – Year 3

Children continue to use arrays and create their own to represent multiplication calculations



$$3 \times 8 = 8 + 8 + 8 = 24$$



$$3 \times 8 = 8 + 8 + 8 = 24$$

Stage 4 – Year 3

Children will continue to use arrays to lead into the grid method of multiplication.

$$14 \times 6$$

The 14 is partitioned (split) into 10 and 4.

The answer to 6×10 is found = 60

The answer to 6×4 is found = 24

The two answers are added together $60 + 24 = 84$

10	4
x	
6	

$(6 \times 10) + (6 \times 4)$

60 + 24

84

10	4
x	
6	
60	24

Stage 5 – Year 3 and 4

In this stage, the array is removed and children use the grid method. This is an important step in retaining children's understanding of multiplication.

$$23 \times 8$$

x	20	3	
8	160	24	160
			+ 24
			<u>184</u>

$$346 \times 9$$

x	300	40	6	
9	2700	360	54	2700
				+ 360
				+ 54
				<u>3114</u>

Year 5 and 6 examples

The grid method can be used for multiplying any numbers, including long multiplication and multiplication involving decimals.

$$4.92 \times 3$$

x	4	0.9	0.02	
3	12	2.7	0.06	12
				+ 2.7
				+ 0.06
				<u>14.76</u>

$$72 \times 38$$

x	70	2	
30	2100	60	2100
8	560	16	+ 560
			+ 60
			<u>2736</u>

Stage 6 – Year 5

The grid method should then be taken into an expanded vertical layout.

368×6				
x	300	60	8	+ 1800
6	1800	360	48	+ 360
				+ 48
				<u>2208</u>

↓

Th	H	T	U
3	6	8	
x	6		
	4	8	(8 × 6)
	3	6	0 (60 × 6)
+ 1	8	0	0 (300 × 6)
	<u>2</u>	<u>2</u>	<u>0</u> <u>8</u>

Stage 7 – Year 5 and 6

The expanded method should then be taken into the compact vertical method.

The place value columns are still labelled to ensure children understand the value of each digit in the original number and the answer.

Th	H	T	U
3	6	8	
x	6		
	4	8	(8 × 6)
	3	6	0 (60 × 6)
+ 1	8	0	0 (300 × 6)
	<u>2</u>	<u>2</u>	<u>0</u> <u>8</u>

→

Th	H	T	U
3	6	8	
x	6		
	4	8	
	3	6	
+ 1	8	0	
	<u>2</u>	<u>2</u>	<u>0</u> <u>8</u>

Stage 8 - Year 5 and 6

The vertical method for long multiplication builds on children being efficient when using grid method. Mental addition of the top and bottom rows separately will help children identify these answers in the vertical method.

x	600	90	3	
20	12000	1800	60	= 13 860
4	2400	360	12	= 2 772 +
				<u>16 632</u>

Again the place value columns are labelled to support children in understanding the value of the digits in the original numbers and in the answer.

Step 1

T	Th	H	T	U
6	9	3		
x	24			
	2	7	2	(693 × 4)
+ 1	3	8	6	0 (693 × 20)
	<u>7</u>	<u>7</u>	<u>2</u>	<u>0</u>

As with other calculations, start with the least significant digit, which means we are doing the equivalent of the bottom row of the grid method from right to left.

Carried digits are crossed out to avoid confusion as the method continues.

Step 2

T	Th	H	T	U
6	9	3		
x	24			
	2	7	2	(693 × 4)
+ 1	3	8	6	0 (693 × 20)
	<u>7</u>	<u>7</u>	<u>2</u>	<u>0</u>

The next step is multiplying by the multiple of 10. This is equivalent to the top row of the grid method.

Therefore, if the answer has 2 digits, this is simply put in the correct place.

Whereas if the answer has 3 digits, the TU digits are put into the answer and the H digit is carried into this column.

Step 3

T	Th	H	T	U
6	9	3		
x	24			
	2	7	2	(693 × 4)
+ 1	3	8	6	0 (693 × 20)
	<u>7</u>	<u>7</u>	<u>2</u>	<u>0</u>

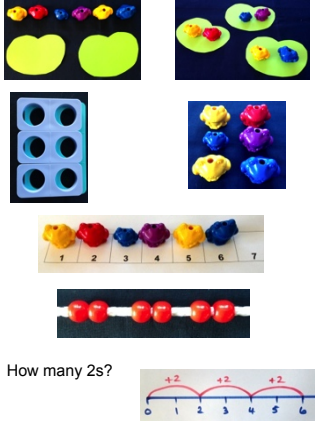

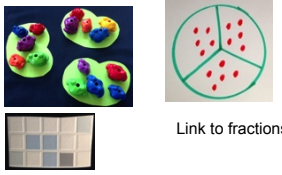

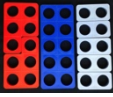

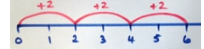
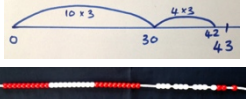
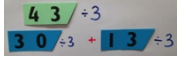

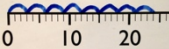
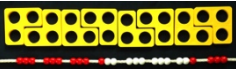

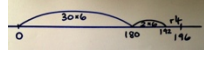


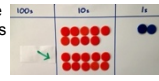
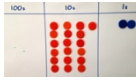
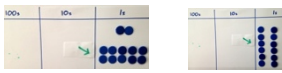
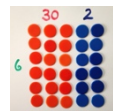
The final step is to add the two answers together.

Children should not be made to go onto the next stage if:

- 1) they are not ready.
- 2) they are not confident.

Children should be encouraged to consider if a mental calculation would be appropriate before using written methods.

Division

Year	1	2	3	4	5	6
Written Methods		Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (\times), division (\div) and equals (=) signs	Write and calculate mathematical statements for \div using the \times tables they know progressing to formal written methods.		Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context $194 \div 6$ $\begin{array}{r} 32 \\ 6 \overline{) 194} \\ \underline{18} \\ 14 \\ \underline{12} \\ 20 \\ \underline{18} \\ 2 \end{array}$	Divide numbers up to 4-digits by a two-digit whole number using the formal written method of short division where appropriate for the context $564 \div 13$ $\begin{array}{r} 43r5 \\ 13 \overline{) 564} \\ \underline{52} \\ 44 \\ \underline{39} \\ 50 \\ \underline{39} \\ 110 \\ \underline{104} \\ 6 \end{array}$ <i>Known multiplication facts:</i> $13, 26, 39, 52, 65, \dots$ $10 \times 13 = 130, 20 \times 13 = 260, \dots$
Developing conceptual understanding	$6 \div 2 = 3$ by sharing into 2 groups and by grabbing groups of 2  How many 2s? 	$15 \div 3 = 5$ in each group (sharing)  Link to fractions $15 \div 3 = 5$ groups of 3 (grouping)  $10 \div 2 = 5$  Use language of division linked to tables  How many 2s? 	Grouping using partitioning $43 \div 3$ If I know $10 \times 3 \dots$   Use language of division linked to tables  How many 3s?  	Grouping using partitioning $196 \div 6$ If I know $3 \times 6 \dots$ then $30 \times 6 \dots$  'Chunking up' on a number line $196 \div 6 = 32 r 4$  Use language of division linked to tables. 	$192 \div 6$ using place value counters to support written method  Exchange one 100 for ten 10s  19 tens into groups of 6  3 groups so that is 30×6 , exchange remaining 10 for ten 1s  So $192 \div 6 = 32$ 	$564 \div 13 = 43 r 5 = 43$ Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context $564 \div 13$ $\begin{array}{r} 43.38 \dots \\ 13 \overline{) 564.00} \\ \underline{52} \\ 44 \\ \underline{39} \\ 50 \\ \underline{39} \\ 110 \\ \underline{104} \\ 6 \end{array}$ $= 43 r 5 = 43 = 43.4$ (to 1dp)
With jottings ... or in your head	Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher	Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts	Write and calculate mathematical statements for multiplication and division using the multiplication tables and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental methods	Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers Recognise and use factor pairs and commutativity in mental calculations	Multiply and divide numbers mentally drawing upon known facts Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000	Perform mental calculations, including with mixed operations and large numbers
Just know it!	Count in multiples of twos, fives and tens	Recall and use \times and \div facts for the 2, 5 and 10 x tables, including recognising odd and even numbers.	Recall and use \times and \div facts for the 3, 4 and 8 times tables	Recall \times and \div facts for \times tables up to 12×12 .	Recall prime numbers up to 19 know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers	
Foundations	Count back in 2s	Division facts (2 x table)	Review division facts (2x, 5x, 10x table)	Division facts (4x, 8x tables) 10 times smaller	Division facts (4x, 8x tables) 100, 1000 times smaller	Division facts (up to 12×12)
	Count back in 10s	Division facts (10 x table)	Division facts (4 x table)	Division facts (3x, 6 x, 12x tables)	Division facts (3x, 6 x, 12x tables) Partition to divide mentally	Partition to divide mentally
	Halves up to 10	Halves up to 20	Halve two digit numbers	Halve larger numbers and decimals	Halve larger numbers and decimals	Halve larger numbers and decimals
	Count back in 5s	Division facts (5 x table)	Division facts (8 x table)	Division facts (3x, 9x tables)	Division facts (3x, 9x tables) 100, 1000 times smaller	Division facts (up to 12×12)
	Halve multiples of 10	Count back in 3s	Division facts (3 x table)	Division facts (11x, 7x tables)	Review division facts (11x, 7x tables) Partition decimals to divide mentally	Partition to divide mentally
	How many 2s? 5s? 10s?	Review division facts (2x, 5x, 10x table)	Division facts (6 x table) or review others	Division facts (6x, 12x tables)	Review division facts (6x, 12x tables) Halve larger numbers and decimals	Halve larger numbers and decimals

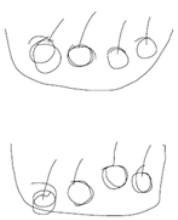
Stage 1 - EYFS

Children are encouraged to develop a mental image of the number system in their heads to use for calculation. They should experience practical calculation opportunities involving **equal groups** and **equal sharing**.



They may develop ways of recording calculations using pictures.

A child's jotting showing halving six spots between two sides of a ladybird.



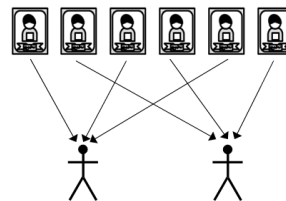
A child's jotting showing how they shared the apples at snack time between two groups.

Stage 2 – Year 1

Children explore practical contexts where they share equally and group equally. $6 \div 2 = ?$

Equal sharing (6 shared equally between 2)

6 football stickers are shared equally between 2 people, how many do they each get? Children may solve this by using a 'one for you, one for me' strategy until all of the stickers have been given out.



Equal grouping (How many groups of 2 are there in 6?)

There are 6 football stickers, how many people can have 2 stickers each?



Stage 3 – Year 2

Children continue to use practical equipment to represent division calculations as grouping (repeated subtraction) and use jottings to support their calculation.

$12 \div 3 = ?$ Children begin to read this calculation as, 'How many groups of 3 are there in 12?'



At this stage, children will also be introduced to division calculations that result in remainders.

$13 \div 4 = 3$ remainder 1



Stage 4 – Year 3

$43 \div 8$



$43 \div 8 = 5$ remainder 3

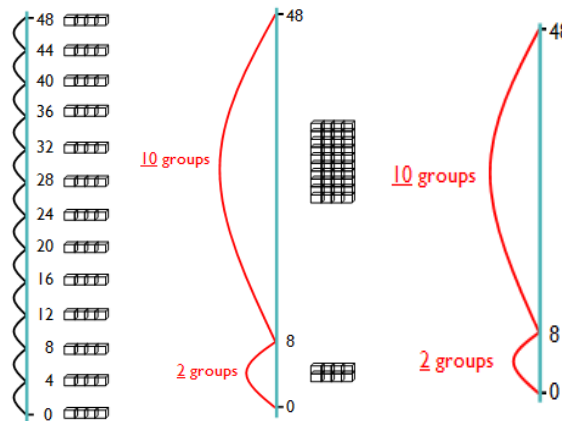
At this stage, children also learn if the remainder should be rounded up or down e.g. $62 \div 8 = 7$ remainder 6

I have 62p. Sweets are 8p each. How many can I buy?
 Answer: 7 (the remaining 6p is not enough for another sweet)
 Apples are packed into boxes of 8. There are 62 apples. How many boxes do I need?
 Answer: 8 (the remaining 6 apples still need to be placed into a box)

Stage 5 – Year 3 and 4

The previous method of repeated subtraction on a number line is continued, but using a vertical number line alongside practical equipment.

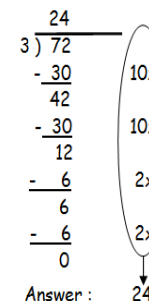
The repeated subtraction is made more efficient by subtracting 'chunks' of the divisor.



Stage 6 – Year 4

This is the 'chunking' method of division in which children use key facts of the multiplication tables of the divisor.

$72 \div 3$



1x	3
2x	6
5x	15
10x	30

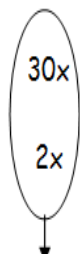
Children should write key facts in a menu box. This will help them in identifying the largest group they can subtract in one chunk.

Stage 7 – Year 5

During this stage children should become more efficient when using the chunking method by not having any subtraction steps that repeat a previous step. For example, when performing $196 \div 6$ an initial subtraction of 60 (10×6) and two further subtractions of 60 (10×6 each) should be changed to a single subtraction of 180 (30×6).

$196 \div 6$

$$\begin{array}{r} 32 \text{ r } 4 \\ 6 \overline{) 196} \\ - 180 \\ \hline 16 \\ - 12 \\ \hline 4 \end{array}$$



1x	6
2x	12
4x	24
5x	30
10x	60
20x	120

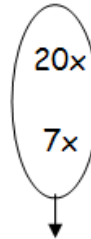
The key facts in the menu box should be extended to include 4x and 20x.

Answer: 32 remainder 4 or 32 r 4

Stage 7 continued – Year 6

$972 \div 36$

$$\begin{array}{r} 27 \\ 36 \overline{) 972} \\ - 720 \\ \hline 252 \\ - 252 \\ \hline 0 \end{array}$$



Answer: 27

Children should not be made to go onto the next stage if:

- 1) they are not ready.
- 2) they are not confident.

Children should be encouraged to consider if a mental calculation would be appropriate before using written methods.